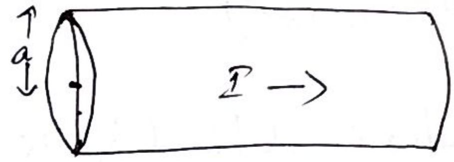
Problem set on Ampere's law:

Q.1 Find the magnetic field of an infinite uniform surface current $\vec{K} = k\hat{x}$, flowing over the xy-plane.

Q.2 A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both inside and outside the wire if,

(a) The current is uniformly distributed over the outside surface of the wire.

(b) The current is distributed in such a way that J is proportional to s, the distance from the axis.



Hint: a) Apply Ampere's law

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B\cdot 2\pi s = \mu_0 I$$

obtain $\vec{B}$ for both the conditions

b) first take $J \propto s$ or $J = \alpha s$, α arbitrary constant

$$\because \quad I = \int_0^a J\, ds$$

$$I = \int_0^a \alpha s\,(2\pi s)\, ds = \frac{2\pi\alpha a^3}{3}$$

$\Rightarrow \alpha = \dfrac{3I}{2\pi a^3}$, Next apply Ampere's law.

Q.3 Find the magnetic field a distance s from a long straight wire, carrying a steady current I, using Ampere's law and the Biot - savart law.

Q.4 Find the vector potential of an infinite solenoid with n turns per unit length, radius R and current I.

Hint:
$$\vec{B} = \nabla \times \vec{A}$$

$$\int_S \vec{B} \cdot d\vec{s} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l} \quad — (1)$$

$$\Rightarrow \phi = \oint_C \vec{A} \cdot d\vec{l} \quad \{ \phi = \text{flux of } \vec{B}$$

{ Stokes' theorem

Apply Ampere's law in integral form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I. \quad — (2)$$

use relation (1) and (2) to obtain vector potential $\vec{A}$.